

Reichenbach's Relativised A Priori

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Abstract

Logical empiricists unanimously criticize the Kantian notion of *a priori* and related epistemic claims. Although cognitively relevant, the notion appeared untenable after the developments in geometry (Bolyai and Lobachevsky, Riemann, Minkowski, and Hilbert, among others) and physics (primarily, Lorentz and Einstein) revolutionized scientific research and its epistemic method. Nevertheless, a clear assessment of the Kantian *a priori* remains problematic because various readings revise or reject its properties, justifying Parrini's (2002) distinction between a *weak* and a *strong* rejection of Kant among logical empiricists. For instance, Schlick (1918), following Poincaré, argues for the conventional nature of the *a priori* by replacing the Kantian notion, based on sensible intuition, with that of implicit definition, primarily justified by Hilbert (1902). On the contrary, the early Reichenbach (1920) relativizes but retains the constitutive function of the *a priori*, dismissing its apodicticity alone. He argues for principles coordinating the theoretical and the empirical, which he sees as equivalent to Kant's synthetic *a priori* judgments. Notwithstanding Reichenbach's (1924, 1928) later acceptance of Schlick's criticism, his notion of relativized *a priori* aims to bridge the gap between abstract mathematical structures and concrete physical phenomena (Friedman, 2001; De Boer, 2010), maintaining the apriority's constitutive feature, also present in Carnap's *L-rules* (1928; Friedman, 1999). This paper shows Reichenbach's weak rejection of the Kantian *a priori* and its cognitive-metaphysical implications.

In light of Lorentz's and Einstein's new physics and its non-Euclidean geometry (Bolyai and Lobachevsky, Riemann, Minkowski, among others), logical empiricists unitedly reject Kant's version of *a priori*, especially its psychological and apodictic features (Friedman, 2007). Nevertheless, although transformed, its cognitive purpose is retained (Parrini, 1998). Instances of logical-empiricist variations of the *a priori* are the notions of (a) *relativized a priori* and related *coordinative definitions* (Reichenbach, 1920; Friedman, 2009), (b) *implicit definitions* (Schlick, 1918; Popper, 1959; Einstein, 1921; Giovannini-Schiemer, 2019), and (c) *L-rules* (Carnap, 1928). While (a) and (c), although in different ways (Friedman, 1999), maintain the constitutive character of the *a priori*, (b) only argues for its conventionality, following Poincaré (1902) and Hilbert (1899). These *weak* and *strong* rejections of the Kantian *a priori* (Parrini, 2022) betray its uneasy evolution through logical empiricism, showing irreducible inconsistency. However, if the *a priori* evolve according to empirical findings, in what sense is it still Kantian?

1. Constitution and apodicticity

Among the features of the Kantian *a priori* (see Anderson, 2014), logical empiricists isolate two. For them, *a priori* means “necessary and unrevisable, true for all time,” and /or “constitutive of the concept of the object of [scientific] knowledge” (Friedman 2001: 72). The distinction appears first in Kant, then occurs in Reichenbach (1920) and Carnap (1928). Kant’s *a priori* principles are absolutely (i.e., non-relatively) necessary or apodictically certain but also perform a “constitutive function with respect to *a posteriori* or empirical truths” (Friedman, 2001: 73). Such function makes the empirical cognition and confirmation of related truths possible. Therefore, *a priori* principles must be unrevisable by default.

The reason that *a priori* knowledge is in fact independent of empirical cognition or experience, for *Kant*, is that *a priori* knowledge yields the necessary conditions under which alone empirical cognition or experience can take place. Since they formulate the necessary conditions or rules for establishing empirical knowledge, *a priori* principles cannot themselves be similarly established; and it is in precisely this sense that they are prior to or independent of experience. (Friedman, 2001: 73)

However, logical empiricists challenge this type of unrevisability or independence from experience. Once they acknowledge that “those principles Kant took to be *a priori* can, after all, be revised,” says Friedman, “the way is then open, as it was for Reichenbach and Carnap, to retain Kant’s characteristic understanding of *a priori* principles as constitutive” (2001: 73) while dismissing the marks of necessity, unrevisability, and apodictic certainty (see Oliva, 2024).

Accordingly, for Reichenbach (1920), exact sciences rely on “the notion of a relativized yet still constitutive *a priori*” (Friedman, 2001: 71), although “*a priori* principles (both mathematical and physical) change and develop with the continual progress of empirical natural science” (Ibid), unlike Kant thought.

According to the traditional conception of the *a priori*, in which it means “justified independently of experience,” [...] any principle correctly characterized as *a priori* would perforce have to hold (if it does hold) entirely independently of all empirical findings and would thus have to hold “come what may”. (Friedman, 2001: 71)

Essential to this notion of relativized *a priori* (i.e., constitutive and revisable) is the relation between *a priori* and *a posteriori* cognition. What exactly does it mean for the *a priori* to represent (a) *necessary conditions* and (b) *constitutive principles* of empirical knowledge? Kant overlaps (a) and (b), holding that constitutive principles are necessary conditions (of the possibility of empirical laws). Still, Friedman warns us that (a) differs from a standard sense, “where A is a necessary condition of B simply if B implies A” (2001: 74). Indeed, in the Kantian sense, (a) entails (b), namely a normative framework for empirical knowledge.

To say that A is a constitutive condition of B rather means that A is a necessary condition, not simply of the truth of B, but of B's meaningfulness or possession of a truth value. It means [...] that A is a *presupposition* of B. (Friedman, 2001: 74)

Consider Newton's physics, where the law of universal gravitation uses the absolute acceleration concept, which doesn't have empirical meaning or application (within that physics) unless the laws of motion hold. So, “we know how to give empirical meaning and application to the law of universal gravitation,” explains Friedman, only “by presupposing that the laws of motion are true” (2001: 75). These laws work as *a priori* principles. So, if they are untrue, there cannot exist the frame of reference in which they hold, entailing that “the question of the empirical truth (or falsity) of the law of universal gravitation,” states Friedman, “cannot even arise” (Ibid).

2. Coordinative principles

For Parrini, the relativized *a priori* bridges the gap “between abstract mathematical structures and concrete physical phenomena” (Friedman, 2001: 78). This issue is central to the logical empiricist agenda. Schlick and Reichenbach identify a special class of non-empirical physical principles to solve it, which they call *coordinating or constitutive principles* and *conventions* (in Poincaré's sense), respectively. Van Fraassen exemplifies this class with the notion of measurements, which help determine the values of mathematical functions. Without them, a theory remains pure. It could never become empirical “if its terms were not linked to measurement procedures” (Van Fraassen, 2008: 115). This linkage raises the problem of coordination: “*How*

can an abstract entity, such as a mathematical space, represent something that is not abstract, something in nature?" (Van Fraassen, 2006: 537).

Reichenbach replies by pointing to modern physics (see Oliva 2024), where mathematical equations represent all processes. Still, the two sciences significantly differ. Indeed, "the truth of mathematical propositions depends upon internal relations among their terms," but "the truth of physical propositions," argues Reichenbach, "depends on relations to something external, on a connection with experience" (1965: 34). Consequently, we ascribe absolute certainty to (1) the former kind of assertions and probability to (2) the latter. But what about (3) their relationship? Let's see (1-3) in detail.

(1) Reichenbach endorses a quasi-structuralism in mathematics, where entities are determined by primitive definitions (i.e., axioms) whose terms rely on the other defining terms belonging to a shared framework.

The *mathematical object* of knowledge is uniquely determined by the axioms and definitions of mathematics. The definitions indicate how a term is related to previously defined terms. The mathematical object receives meaning and content within this framework of definitions through an analysis of its differences from and equivalences to other mathematical objects. (Reichenbach, 1965: 35).

Accordingly, the axioms present the mathematical rules for defining concepts (see Oliva, 2024). All concepts, including the fundamental ones (i.e., those occurring in the axioms themselves), are defined through relations. Reichenbach justifies his argument by referring to Hilbert. Consider his axiom of order II-3, stating that "Of any three points situated on a straight line, there is always one and only one which lies between the other two" (Hilbert, 1902: 4). Here, Hilbert describes the properties of 'point,' 'straight line,' and 'between' through a *non-exhaustive* definition, made complete solely by the totality of the axioms. All the entities involved (i.e., 'point,' 'straight line,' and 'between') have the axiom-stated properties, owing their nature to mutual relations that can change. For, in projective geometry, 'straight-line' and 'point' are interchanged, preserving the truth of related theorems since "their axiomatically defined relations are symmetrical for the two concepts" (Reichenbach, 1965: 35), although, as Schlick noticed (1974), our

intuition depicts the two concepts dissimilarly, ascribing different contents to the axioms.

(2) For Reichenbach, the method of representing physical events relies on mathematical equations. It defines one magnitude in terms of others by relating them to increasingly general magnitudes, up to the axioms. “Yet what is obtained,” he argues, “is just a system of mathematical relations,” ultimately lacking a statement of its significance, namely “the assertion that the system of equations is *true for reality*” (1965: 36).

The *physical object* cannot be determined by axioms and definitions. It is a thing of the real world, not an object of the logical world of mathematics. (Reichenbach 1965: 36)

(3) Hence, the internal coherence of mathematics doesn't suffice for physical truths, which further must entail a precise relation between equations and physical phenomena.

The physical relation can be conceived as a coordination: physical things are coordinated to equations. Not only the totality of real things is coordinated to the total system of equations, but *individual* things are coordinated to *individual* equations. The real must always be regarded as given by some perception. (Reichenbach, 1965: 36-7)

E.g., to name the earth a sphere, we must coordinate the geometrical spherical figure to a specific visual perception, i.e., a perceptual image of the earth, according to some primitive coordination principles (see Oliva, 2024). For this purpose, Reichenbach refers to Boyle's gas law, where we coordinate the formula $p \times V = R \times T$ to direct (e.g., feelings) and indirect (e.g., the position of a monometer's pointer) perceptions of gas. Indeed, “our sense organs mediate between concepts and reality” (Reichenbach, 1965: 37) – today's formula for the ideal gas is $PV = nRT$. Such coordination occurs in two moments. Accordingly, within the context of any particular scientific theory, Reichenbach identifies two types of cognitive principles: (a) the *axioms of connection* defining empirical laws that involve already well-defined terms and concepts, and (b) the *axioms of coordination*, namely non-empirical principles laid down antecedently to ensure that empirical well-definedness.

The peculiar nature of such coordination allows us to establish a correspondence between two sets by coordinating every element of one set with an element of the other. For this purpose, “the elements of each set must be defined,” argues Reichenbach, i.e., “for each element, there must exist another definition in addition to that which determines the coordination to the other set” (1965: 37). Yet, although the ‘equations’ (i.e., the conceptual side of the coordination) are uniquely defined, the ‘real’ (i.e., the side dealing with the cognition of reality) isn’t. Reichenbach overcomes this issue as follows.

The definition results from coordinating things to equations. Thus, we are faced with the strange fact that in the realm of cognition, two sets are coordinated, one of which not only attains its order through this coordination but whose elements are *defined by means of this coordination*. (Reichenbach 1965: 40)

Therefore, Reichenbach’s coordinating principles define reality according to our perceptions and mathematical equations. In this regard, they resemble Kant’s conceptualization of sensible intuitions, which applies rules on sensations, namely schemata and pure principles of understanding.

3. Synthetic a priori principles

Do Kant’s constitutively *a priori* schemata, mediating between algebraic abstractions and applied physics, anticipate Reichenbach's coordination?

Like Reichenbach’s, Kant’s actual object of cognition derives from the employment of formal structures on blind modifications of sensibility, namely raw sensations (see Oliva, 2024). Accordingly, the matter of the cognitive object represents a yet-to-be-determined empirical condition, *a posteriori*. Kant conceives of such determination in two steps, mainly focusing on its possibility or formality. (1) Categories unify the manifold intrinsic to our inner sense, shaping the corresponding *schemata*. Indeed, for the content of a category per se is initially derived from the logical structure of judgments alone, “it must be made applicable to objects whose form has thus far been specified solely by the pure forms of space and time,” say Guyer and Wood (1998: 10). So, schemata associate categories to a form or relation in intuition, particularly an inner temporal one. (2) The *principles of pure understanding* define the rules

for applying these schemata to empirical judgments based on our spatial outer sense. Accordingly, “the *use* of [those] schemata in turn depends upon judgments about the *spatial* properties and relations of at least some objects of empirical judgment” (Ibid).

The *analytic of principles* will accordingly be solely a canon for the *power of judgment* that teaches it to apply to appearances the concepts of the understanding, which contain the condition for rules *a priori*. (Kant 1787: A132/B171)

“Just like Kant’s synthetic *a priori* principles, principles of coordination assign conceptual structures to the realm of experience,” says De Boer, and “bridge the gap between the conceptual and the sensible” (2010: 517). For Reichenbach, “they ultimately define real objects and real events;” therefore, “we may call them constitutive principles of experience” (1965: 49). So, he refers to Kant’s schemata.

Therefore, Reichenbach’s early works evidence a weak rejection of the Kantian *a priori*. As De Boer argues, he “aimed to transform rather than abolish Kant’s notion of synthetic *a priori* principles” (2010: 508); for her, the differences with Kant have been overestimated by logical empiricists and their readers, such as Friedman.

Friedman weds Kant’s transcendentalism to Newton’s mechanics and Euclid’s geometry. If correct, Einstein’s new physics demands a detachment from the Kantian *a priori*, stemming from an obsolete paradigm. However, De Boer separates Kant’s synthetic *a priori* principles from Euclidian geometry and Newtonian physics, which instantiate but don’t demonstrate the validity of those principles. Instead, she thinks such validity has a metaphysical nature, traceable back to Leibniz and Hume. Friedman believes that Kant abandoned classic metaphysics (see 1992: 37-8), but De Boer contends that he seeks to reconcile metaphysics with Newton’s scientific paradigm. However, these two don’t overlap since the first can ground any science without restriction.

De Boer convincingly argues that the conditions constructing (*a*) an object of cognition and those building (*b*) a physical law differ. It’s always the case that (*a*) entails (*b*), but the opposite doesn’t hold.

[...] the synthetic *a priori* principles treated in the *Critique* merely constitute necessary rules for determining the spatio-temporal, law-governed relations between given representations - whatever the actual content of these relations may be. (De Boer 2010: 510)

“Unlike laws of physics [*b*], the principles of pure understanding do not depict the world, but constitute the ‘rules of the pure thinking of an object’ [*a*]” (De Boer, 2010: 510). These principles offer “perspectives that we must necessarily adopt,” says de Boer, “to turn phenomena into objects of knowledge” (Ibid). “Without such synthetic *a priori* root-principles, it would not be possible for us to establish laws of physics proper” (Ibid). So, De Boer sharply divides (*a*) and (*b*). Accordingly, “the synthetic *a priori* principles of pure understanding” represent “the root-principles by dint of which something can become an object of knowledge in the first place,” namely, they delimit the domain in which “something can be treated as an object” (Ibid). However, how does (*a*)’s grounding of (*b*) work?

Thus, far from telling us something about the world, the principle based on the pure concept of quantity merely states the rule that every intuited object has an extension and, hence, can be determined mathematically. According to Kant, it’s only on the basis of this principle that physics can apply pure mathematics to objects of experience (A165/B206). The category of substance, for its part, yields the rule that scientific knowledge must necessarily distinguish between that which changes over time and that which constitutes the self-identical substrate of such changes. Otherwise, scientists would neither be able to determine something as an object, nor to determine the relation between objects. (De Boer, 2010: 512).

De Boer’s analysis supports the weak rejection of the Kantian *a priori* by showing its similarity with Reichenbach’s coordinative principles. Although relativized, the *a priori* constitutive component must be retained as it grounds every cognitive object before being employed in a scientific theory.

4. Coordination and reality

Unlike Kant, Reichenbach holds that “the content of every perception is far too complex to serve as an element of coordination” (1965: 40). Before

coordination, we must sort out relevant from irrelevant aspects of our perception; namely, we must establish order among them. However, “such a coordination presupposes the equations, or the laws expressed in them” (Ibid). So, Reichenbach maintains that physical knowledge relies entirely on coordination. Indeed, “only a cognitive judgment,” he argues, that is an act of coordination, can decide whether the sensation of a tree corresponds to a real tree” (1965: 41), and not to a hallucination.

So, perceptions don't define what is real. Therefore, the elements of the universal set remain undefined since “one side of the cognitive process contains an undefined class” (1965: 42).

Thus, it happens that individual things and their order will be defined by physical laws. The coordination itself creates one of the sequences of elements to be coordinated. (Reichenbach 1965: 42)

Hence, coordination occurs between a given set of mathematical equations and a completely undetermined reality. Only assigning these equations to experience turns this latter into a proper domain of physics. Indeed, coordinating principles define the individual elements of reality and, in this sense, constitute the real object. Therefore, like Kant, Reichenbach holds that physics relies on rules unifying pure thought (exemplified by mathematics) and sensible experience.

Nevertheless, the two sides of knowledge maintain a mutual relationship. Indeed, “the defined side does not carry its justification within itself,” argues Reichenbach, as “its structure is determined from outside” (1965: 42). So, the coordination to undefined elements is restricted by experience, not arbitrary. Therefore, knowledge preserves an empirical determination or character.

We notice the strange fact that it is the defined side that determines the individual things of the undefined side, and that, vice versa, it is the undefined side that prescribes the order of the defined side. *The existence of reality is expressed in this mutuality of coordination.* (Reichenbach 1965: 42)

Therefore, this mutuality attests to what is real. It also guarantees truth, which consistently derives from correct coordination, correlating to experience data.

As Reichenbach states, “contradictions are discovered by observation” (1965: 43). To be true, a theory must continuously lead to consistent coordination. Like Schlick, Reichenbach consistently defines “*truth in terms of unique coordination*” (Ibid). So, perceptions play a crucial role in the cognitive process since they “*furnish the criterion for the uniqueness of the coordination,*” he claims (1965: 44). Similarly, About the correspondence of judgments with facts, Schlick states that, “a judgment that *uniquely designates* a set of facts is called *true*” (1974: 60).

Despite the differences, Reichenbach underlines the continuity with Kant. His theory of cognitive coordination straightforwardly answers Kant’s question, “How is pure natural science possible?” (B20). In a Kantian fashion, ‘possible’ has a logical, not a psycho-physical, meaning; “it pertains to the logical conditions of a coordination” (Reichenbach, 1965: 47). Hence, Reichenbach rephrases Kant’s question as follows, “*By means of which principles will a coordination of equations to physical reality become unique?*” (1965: 48). Accordingly, these epistemological principles of coordination “are equivalent to Kant’s synthetic a priori judgments” (Ibid).

5. Conclusion

Hence, revisability and sensible dependency define Reichenbach’s ‘weak rejection’ in 1920. Later (1924, 1928, 1936), he softened, without ever endorsing it, though, his early criticism of Schlick’s ‘strong rejection’ of Kant’s *a priori*, based on the conventionalist characterization of this latter and the denial of its constitutive function. The idea that constitutive scientific principles can be *a priori* and revisable at the same time perfectly instantiates Parrini’s notion of ‘weak rejection,’ balancing continuity and discontinuity with Kant.

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