

# Mathematical Intuitionism

Prof. Luca Oliva

## Course Description

The course introduces and analyses some issues of epistemology and philosophy of mathematics that fall under the label 'intuitionism'. Our overview of intuitionism starts with the classic introductions of Shapiro and Pust, who outline the notion of intuition in mathematics and epistemology, respectively. Hence emerges the significance of the notion for the mathematical and the philosophical practice. Statements such as "if not-not- $p$ , then  $p$ " represent a source of evidence for further statements. The former statements are usually called intuitions. In this sense, all logical tautologies, natural deduction, and algebraic axioms are intuitive. Logical justifications and mathematical foundations equally rely on such evidence. The course will first explore the turning point of intuitionism represented by the Kantian account, where mathematical statements are reduced to intuition-based constructions. This part refers to the readings of Hintikka, Parsons, Posy, and Maddy. From the Kantian account, Brouwer seems to derive the intuition of two-oneness, the basal intuition of his mathematics, which creates not only the numbers one and two, but also all finite ordinal numbers. Our analysis of Brouwer's algebraic intuitionism will include Heyting and Dummett. In relation to Brouwer, we will also consider the perceptual intuitionism of Gödel and Hilbert in the readings of Burgess and Parsons, and Tieszen's account of Husserl's phenomenological intuition. Common to all mathematical intuitionists is the idea that a mathematical statement corresponds to a mental construction, and a mathematician can assert the truth of a statement only by verifying the validity of that construction by intuition. The last two parts of our course are devoted to epistemic intuitions. In epistemology, intuitions offer the ultimate evidence or justification for our theories (Chudnoff). However, lately, some scholars such as Cappellan were influenced by deflationist readings of Lewis and Williamson, who reduce intuitions to beliefs or dispositions to believe. We will finally devote our attention to their arguments.

## Schedule of Class

2 5	4:45 – 6:15 (3C)	Introduction to Intuitionism (Pust, Schapiro)
9 5	4:45 – 6:15 (3C)	Mathematical and Logical Instances (Set Theory)
16 5	4:45 – 6:15 (3C)	Kantian Intuitionism (Kant)
23 5	4:45 – 6:15 (3F)	Kantian Intuitionism (Hintikka, Parsons, Posy, Maddy)
	6:30 – 8 (3F)	Phenomenological Intuition (Sokolowsky, Zahavi)
29 5	4:45 – 6:15 (3B)	Phenomenological Intuition (Tieszen, Husserl)
6 6	4:45 – 6:15 (3C)	Mathematical Intuitionism (Brouwer, Heyting)
	6:30 – 8 (3F)	Mathematical Intuitionism (Dummett)
13 6	4:45 – 6:15 (3C)	Quasi-Intuitionism (Wittgenstein)
27 6	4:45 – 6:15 (3C)	Deflationist Theories (Cappellan, Chudnoff, Williamson)

## Assignments and Assessments

Here is the list of assignments and their corresponding value. (1) Attendance and participation, tests (20%), (2) a summary of one class subjects (20%), (3) an argumentative and comprehensive final paper (4000 words) on the theoretical parts of the class (60%).

Plagiarism or other forms of cheating will be reported and penalized. Look at the University of Vienna bylaws for details on academic honesty, disabilities, and other matters about students and life on campus.

## Bibliography

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