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Non-Analytic Necessity in Pap

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Abstract

In 1944, Pap argues for different kinds of apriority, distinguishing between *formal*, *material*, and *functional* a priori. This paper focuses on formal apriority and examines Pap's arguments against reducing necessary sentences to analytic sentences. First, it explores the background of Pap's semantics, consisting of the works on analyticity and apriority by Leibniz, Kant, Ayer, and Wittgenstein among others. Next, it outlines Pap's criticism of the Carnapian notion of analyticity based on syntactic concepts, emphasizing the relevance of semantic criteria of adequacy (Pap 1946). Following that, the paper analyzes Pap's cases of non-analytic necessity (Pap 1949), which include *temporal statements* such as "$(\forall x)(\forall y)[xRy \supset \sim yRx]$," relying on asymmetry, as well as the *color-exclusion problem*, expressed by statements like "$\sim(\exists x)(\exists t)(\text{blue}_{xt}.\ \text{red}_{xt})$" or "$\forall x(\text{Red}(x) \rightarrow \neg\text{Blue}(x))$." Finally, it enhances Pap's analysis by comparing it with Husserl's concept of *material* a priori. Supporting references include works by Shieh (2006), Stump (2011, 2021), Mormann (2021), and Limbeck-Lilienau (2025).

Keywords

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In 1944, Pap examined various types of "a priori" beyond the scientific statements that were typically used as standard references by logical empiricists when criticizing the Kantian model. He specifically identified three types of apriority: *formal*, *functional*, and *material*, each of which entails three types of necessity.

First, the *formal* or *analytic* a priori represents a formal or logical necessity, characteristic of logical truths. These truths can be understood as “tautologies” (as suggested by logical positivists) or as “truths of reason” (as defined by Leibniz). Second, the *functional* a priori indicates a *functional necessity*, including Aristotle’s version of hypothetical necessity. This type of necessity relates to the conceptual means in connection with the objectives or ends of inquiry. Its roots lie in American pragmatism as theorized by C.I. Lewis and Dewey, along with Poincaré’s conventionalism (see Stump 2011; Poincaré 1905). Finally, the *material* a priori denotes *psychological necessity*. Still, the key proponents of this kind of necessity, traditionally marked by self-evidence and the inconceivability of the opposite, explicitly reject psychologism in logic. This rejection is particularly evident in Husserl’s *Logische Untersuchungen* (1900–1901) and Meinong’s *Gegenstandstheorie* (1904).

Beginning with a brief overview of Pap’s philosophical tenets in Part I, this paper[1] focuses on the first type of a priori, centered on the concepts of *necessity* and *analyticity*. In Part II, I’ll present these concepts before Pap, as understood by Leibniz, Kant, and Ayer, who serves as a representative of logical empiricism. In Part III, I’ll discuss Pap’s arguments against reducing necessity to analyticity. Parts IV and V will examine Pap’s instances of non-analytic necessity, including the semantic structure of *temporal sentences* and issues related to the *color-exclusion problem*. Finally, in Part VI, I’ll bolster Pap’s reasoning by comparing these instances with Husserl’s concept of *material* a priori.

21.1. The Basics of Pap’s Semantic Analysis

Recent studies by Stump (2021) and Mormann (2021) have highlighted the neo-Kantian elements found in Pap’s philosophy of science, particularly those derived from Cassirer’s philosophy of physics (see Pap 1946/1962). His scientific work also incorporates aspects of pragmatism, conventionalism, and logical empiricism. However, his semantic analysis is firmly rooted in two philosophical traditions: *logical empiricism* and the *philosophy of language* (see Shieh 2006).

The first of these traditions influences Pap’s perspective on the relationship between philosophical inquiry and the natural sciences. He recognizes a significant contrast between traditional philosophy, which often resembles a battlefield of endless and unresolved disputes, and the natural sciences, which represent a realm of continuous progress, albeit characterized by revolutions and paradigmatic changes. In this context, empirical sciences serve as the standard of genuine knowledge, while metaphysical theories that fall outside the realm of natural sciences should be approached with skepticism. To understand the shortcomings of traditional metaphysics, logical empiricists developed the verificationist theory of meaning, which Pap adopts and furthers.

The principle of verification is a formulation of the shared and objective standards governing the acquisition of knowledge in the sciences: a statement is accepted only on the basis of sensory experiences that establish it as true. […] The cognitive meanings of statements are given by the sensory experiences that would verify or falsify them; a statement for which there is no method of verification has thus no objective meaning at all. […] Thus, science deals only with cognitively significant statements, while metaphysics trafficked in (cognitive) nonsense. (Shieh 2006: 4).

Pap significantly relies on the verifiability principle, despite the issues concerning the apodictic nature of logical laws, which the principle doesn’t adequately explain. While logical empiricists endorse the solution proposed by Wittgenstein, Pap distances himself from this logicist approach. Wittgenstein, along with Ayer and the entire Vienna Circle, contends that the axioms and theorems of mathematics lack justification from experience due to their normative nature. According to their understanding of necessity, logical-mathematical truths define the meanings of our words by establishing the rules governing language use. For all logical empiricists, these meanings or rules are conventional, as their adoption isn’t constrained by logic or experience. In their view, references to experience can be disregarded. For instance, Carnap argues that “meaning rules *cannot be* constrained by empirical reality, because it is only when the rules are in place that we have a conception of what it is for our acceptance of statements to be responsible to empirical evidence” (Shieh 2006: 5).

Wittgenstein’s solution is also embraced by ordinary language philosophy. However, Pap aligns himself with philosophers of language[2] and rejects the central assumption of logical empiricism, which posits that science represents the ultimate paradigm of knowledge and philosophical methodology. He argues that the necessity grounded in linguistic conventions exposes the nonsensical nature of metaphysical claims. The conventional features of our language generate rules—what Wittgenstein (1931–1934) terms "philosophical grammar"—that determine what makes sense when discussing the truth underlying reality. For Pap, these rules "express stipulations concerning the use of symbols and thus cannot significantly be said to be refuted by facts" (1953: 308). In contrast, the sentences governed by these rules "are empirically refutable statements 'about reality'" (Ibid.).

However, analytic truths present a dilemma because a linguistic rule is neither true nor false; instead, it defines what it means for something to be considered true or false. Therefore, semantic rules behave as standards for thinking, which, although seen as conventional, still necessitate a priori agreement. Pap carries out his analysis of *analyticity* and *apriority* accordingly.

To better appreciate Pap's contributions to semantics, let's take a closer look at the traditional conceptions of these concepts.

21.2. Apriority and Analyticity Before Pap

Pap's analysis of formal apriority, along with the related concepts of analyticity and necessity, are primarily (though not exclusively) drawn from the arguments of Leibniz, Kant, Ayer, and Husserl. These references provide the essential framework of Pap's work. So, to fully appreciate Pap's contributions, it's helpful to first examine these references.

21.2.1. Leibniz's a Priori Truths

For Leibniz, all truth derives from the analysis of identities whose components or conceptual classes (*S*-term and *P*-term) are related by inclusion, i.e., containment. Otherwise, "there would be a truth that couldn't be proved a priori, that is, a truth that couldn't be resolved into identities, contrary to the nature of truth, which is always an explicit or implicit identity" (*Primary Truths*, 1686/1989: 31).

> There are also two kinds of *truths*: those of *reasoning* and those of *fact*. The truths of reasoning are necessary, and their opposite is impossible. Those of fact, however, are contingent, and their opposite is possible. When a truth is necessary, we can find the reason by analysis, resolving the truth into simpler ideas and simpler truths until we reach those that are primary. (Leibniz, *Monadology* 1714/1989: § 33)

In true propositions, "the predicate or consequent is always in the subject or antecedent" (1686/1989: 31). "In identities, the connection of the predicate with the subject (its *inclusion in* the subject) is explicit; in all other [true] propositions it is implicit and has to be shown through the analysis of notions" (Ibid.). Therefore, Leibniz assumes (but doesn't prove) that all truths are reducible to identities, meaning that all true propositions are ultimately instances of identity. "First truths are the ones that assert something of itself or deny something of its opposite" (30). Here are some examples:

1 A is A

2 A is not not-A

3 If it's true that A is B, then it's false that A isn't B (namely, it's false that A is not-B)

From these *first* or *primary truths* we can derive truths such as "the whole is greater than its part," or "the part is less than the whole," which are "something easily demonstrated from the definition of 'less' or 'greater,' with the addition of the primitive axiom, that is, the axiom of identity," because "the *less* is that which is equal to a part of the other (the *greater*)" (1686/1989: 31). These claims are clearly valid only for finite classes, but investigating this restriction isn't our current purpose.

As a partial conclusion, Leibniz holds that analyticity implies apriority in two ways. *(a)* Reducing all truths to the first ones or resolving them into their simpler components gives an a priori proof, namely a proof that doesn't depend on experience. However, the independence advocated by Leibniz remains vague. It could be independence from *further* or *all* experience. The proof could thus be *relatively* or *absolutely* a priori, respectively. Leibniz offers no clarification on this aspect. *(b)*

Despite this vagueness, it's clear that, for Leibniz, a priori demonstrations rest on the analysis of the propositional terms. The same holds for both derived and primary truths.

21.2.2. Kant's Formal a Priori

Kant extends Leibniz's concept of necessity to include non-analytic instances, ultimately based on what he calls "synthetic apriority." Along with this development, he revolutionizes the concept of apriority. With the term "a priori," Kant primarily designates *(1)* the *existential and cognitive conditions* of every object and *(2)* a *modal proposition* about necessity. Consistently, the logical-empiricist analysis identifies two central properties—"constitution" and "apodicticity"—characterizing the Kantian notion of a priori (see Friedman 2001/2007).

1. The a priori performs a constitutive function. It connotes all the conditions (of possibility) determining our cognition and its objects before sensation. Accordingly, every object of cognition must be determinable a priori. As a lemma, the a priori is mind-dependent: it follows that the formal structure of any cognitive object is established a priori. So, it belongs to our mental faculties.

2. The a priori characterizes all necessary judgments. Kant distinguishes between *analytic* and *synthetic* judgments. Roughly put, all analytic judgments are necessary since they are valid a priori, namely they are justifiable independently of experience. E.g., "all bodies are extended" (A7/B11). On the contrary, synthetic judgments can be a priori, therefore necessary, or a posteriori, therefore unnecessary. Their respective instances are "7 + 5 = 12" (B15) or "space has three dimensions" (A239/B299), and "all bodies are heavy" (A7/B11). The a priori - a posteriori distinction has an epistemological connotation and pertains to the justification of the validity of judgments. So, for Kant, the validity of a priori judgments is independent of experience and is, therefore, *universal* and *necessary*. In contrast, the validity of a posteriori judgments depends on experience and is *particular* and or *contingent*.

Further, Kant holds that a judgment can be relatively or absolutely a priori. In the latter case, the judgment isn't *independent of a particular experience* but *absolutely independent of all experience*. In the former case, the judgment is empirical and precedes another judgment. Consistently, Kant opposes unrestricted to restricted necessity.

Besides a priori and a posteriori, Kant also defines judgments as *synthetic* and *analytic*, one of the most problematic characterizations for logical empiricists.

> He [Kant] said that an analytic judgement was one in which the predicate B belonged to the subject A as something which was covertly contained in the concept of A. He contrasted analytic with synthetic judgements, in which the predicate B lay outside the subject A, although it did stand in connection with it. Analytic judgements, he explains, "add nothing through the predicate to the concept of the subject, but merely break it up into those constituent concepts that have all along been thought in it, although confusedly." Synthetic judgements, on the other hand, "add to the concept of the subject a predicate which has not been in any wise thought in it, and which no analysis could possibly extract from it." Kant gives "all bodies are extended" as an example of an analytic judgement, on the ground that the required predicate can be extracted from the concept of "body," "in accordance with the principle of contradiction"; as an example of a synthetic judgement, he gives "all bodies are heavy." (Ayer 1952: 77)

Kant defines analyticity consequently. Two of its properties are especially relevant. *(1)* In analytic judgments, the predicate's connection is "thought through identity" (A7/B10). If this connection is thought without identity the judgments are instead synthetic. *(2)* Analytic judgments clarify, and synthetic judgments amplify our cognition. Indeed, holds Kant, "through the predicate the former [judgments] do not add anything to the concept of the subject, but only break it up by means of analysis into its component concepts, which were already thought in it (though confusedly)" (Ibid.). On the contrary, the latter [judgments] "add to the concept of the subject a predicate that was not thought in it at all and could not have been extracted from it through any analysis" (Ibid.).

(1) Connection without identity (i.e., synthesis) and *(2)* partition of identities (i.e., analysis) imply *(3)* containment, on whose notion analyticity relies (see Anderson 2015).Kant conceives "subject" and "predicate" as classes containing specific parts or marks, i.e., members. In analytic propositions, we don't need to go outside the *S*-term to find the *P*-term. We need instead to analyze the *S*-term to encounter the *P*-term therein. For Kant, this analysis awakens our awareness of the manifold we always think in the *S*-term. So, it unsurprisingly bears a psychological connotation. In contrast, synthetic propositions show no containment. In them, the *P*-term is something entirely different from the *S*-term and lays outside of it. Therefore, the relation between the two terms (i.e., concepts or classes) is possible in two ways, namely as "containment (for analytic judgments), and connection without containment (for synthetic ones)" (Anderson 2015: 8).

Besides that, Kant also discusses a third kind of judgment, the synthetic a priori. In this case, the help from experience is entirely lost. Hence, how do we combine concept *A* with concept *B* lying outside *A*? How can we justify their synthesis?

> What is the X here on which the understanding depends when it believes itself to discover beyond the concept of *A* a predicate that is foreign to it and that is yet connected with it? It cannot be experience, for the principle that has been adduced adds the latter representations to the former not only with greater generality than experience can provide, but also with the expression of necessity, hence entirely *a priori* and from mere concepts. (Kant 1787: A9/B13)

Kant's solution involves the basics of his theory of cognition, which starts with an affection that modifies our sensibility. If the perceived alteration is ignored, sensible intuition and the spatiotemporal field where it occurs still remain. This intuition stands for a part of that spatiotemporal whole. Since it carries nothing empirical, it provides pure content to which concepts may apply. So, the same operation synthesizing sensation-bearing (i.e., empirical) intuitions of an apple can also unify them when they are still sensation-free, namely pure or before all experience. The conceptual (i.e., categorical) synthesis of pure intuitions forms the semantic terms or classes for a priori judgments (i.e., cognitions). In this case, the synthetic operations on terms are entirely possible a priori, as are the formal terms themselves, which are constituted by pure intuitions.

21.2.3. Ayer's Analytic Truths

Like Kant, empiricists eschew metaphysics "on the ground that every factual proposition must refer to sense-experience" (Ayer 1952: 71). However, they differ in that they maintain the validity of a proposition is determined by observation *alone*, rather than solely by logical considerations. As Hume claims, "no general proposition whose validity is subject to the test of actual experience can ever be logically certain" (72). Its verification always allows for confutation at some point in the future. Therefore, a law can only be substantiated in *n* -1 cases, offering no logical guarantee of confirmation in the n^{th} case as well, regardless of how large *n* may be. If this is correct, propositions referring to a *matter of fact* can never be necessarily or universally true. Ayer extends Hume's conclusion to all general propositions with *factual content*. They can at best be a probable hypothesis since none of them ever achieves logical certainty. Nonetheless, these premises fail to account for the knowledge of necessary truths.

Ayer contends that for a rationalist like Kant, thought is an independent source of knowledge. According to his reading of Kant, all the necessary truths about the world are thinkable independently of experience. While the correct assessment of Kant isn't our current purpose, Ayer overlooks that Kantian concepts, including related schemata and rules of synthesis, do require sensible a priori conditions to provide a priori cognitions. Despite this, Ayer's counterargument rejects Kant's a priori justification of necessary truths by stating that these truths aren't necessary or aren't truths about the world. Accordingly, no cognitive facts about the world are independent of sensible experience. Ayer famously states a principle of logical empiricism, specifically targeting Kant's rationalism, "a sentence says nothing unless it is empirically verifiable" (1952: 73). So, against rationalism, Ayer claims that "there are no 'truths of reason' which refer to matters of fact" (Ibid.).

The disagreement with Kant precisely pertains to the nature of logical-mathematical propositions. For Ayer, propositions expressing a priori truths are all analytic. Following Kant's justification of analyticity, he assumes that the latter isn't a source of knowledge per se. Consistently, Ayer concludes that a priori truths aren't cognitive since all analytic propositions stand for *tautologies*—a conclusion Pap rejects.

> For while it is true that we have *a priori* knowledge of necessary propositions, it is not true, as Kant supposed, that any of these necessary propositions are synthetic. They are without exception analytic propositions, or, in other words, tautologies. (Ayer 1952: 84)

Hence, Ayer ultimately justifies analyticity with the notion of tautology, which replaces the synthetic a priori of Kant. Accordingly, he argues that all certain and necessary truths, such as purely mathematical and geometrical propositions, are reducible to tautological reasoning.

> The principles of logic and mathematics are true universally simply because we never allow them to be anything else. And the reason for this is that we cannot abandon them without contradicting ourselves, without sinning against the rules which govern the use of language, and so making our utterances self-stultifying. In other words, the truths of logic and mathematics are analytic propositions or tautologies. (Ayer 1952: 77)

Interestingly, Ayer agrees with Kant's analytic-synthetic distinction but disagrees with its justification. His disagreement is crucial for understanding Pap's reasoning regarding the nature of necessary sentences, as discussed below. For Ayer, a sentence is analytic based on its form, which is determined by the functions of the words rather than the containment of terms. For example, consider *(a)* some ants have established a system of slavery, and *(b)* either some ants are parasitic, or none are. We cannot determine whether *(a)* is true or false by examining the definitions of its constituent symbols; instead, we must rely on observation. Thus, *(a)* is synthetic. In contrast, observation is unnecessary for establishing the truth value of *(b)*.

> If one knows what is the function of the words "either," "or," and "not," then one can see that any proposition of the form "Either *p* is true *or* p is not true" is valid, independently of experience. Accordingly, all such propositions are analytic. (Ayer 1952: 79)

Distancing from Kant, Ayer adopts Wittgenstein's logical conventionalism.

> They [i.e., necessary and certain propositions] simply record our determination to use words in a certain fashion. We cannot deny them without infringing the conventions which are presupposed by our very denial and so falling into self-contradiction. And this is the sole ground of their necessity. (Ayer 1952: 84)

Therefore, as Wittgenstein argues, "our justification for holding that the world could not conceivably disobey the laws of logic," states Ayer, "is simply that we could not say of an unlogical world how it would look" (1952: 84). Conceivably, we might employ different linguistic conventions from those we actually do. Notwithstanding, "whatever these conventions might be, the tautologies in which we recorded them would always be necessary," says Ayer, "for any denial of them would be self-stultifying" (Ibid.).

Ayer consistently explains the apodeictic certainty of logic and mathematics through conventional synonymity, i.e., the semantic relation among words carrying the same meaning in a given context. So, "Our knowledge that no observation can ever confute the proposition '7 + 5 = 12' depends simply on the fact that the symbolic expression '7 + 5' is synonymous with '12' (Ayer 1952: 85). Most importantly, "the same explanation holds good for every other a priori truth" (Ibid.).

Invention and discovery are consistent with the tautological nature of mathematics. Poincaré criticizes this view, but his remarks nevertheless exemplify it.

> If all the assertions which mathematics puts forward can be derived from one another by formal logic, mathematics cannot amount to anything more than an immense tautology. Logical inference can teach us nothing essentially new, and if everything is to proceed from the principle of identity, everything must be reducible to it. But can we really allow that these theorems which fill so many books serve no other purpose than, to say in a roundabout fashion 'A=A'? (Poincaré 1905: 1–2 = Ayer 1952: 85)

For Ayer, Poincaré underestimates the limits of our reason, unable to see at a glance everything or learn anything from still unconscious logical inferences. So, even a simple tautology, such as "91 • 79 = 7189," lies beyond the scope of our immediate apprehension. To assure the synonymity of "7189" with "91 • 79" we must compute. However, the calculation "is simply a process of tautological transformation," says Ayer, "by which we change the form of expressions without altering their significance" (1952: 86).

> The multiplication tables are rules for carrying out this process in arithmetic, just as the laws of logic are rules for the tautological transformation of sentences expressed in logical symbolism or in ordinary language. (Ayer 1952: 86)

Therefore, the only satisfactory explanation of the a priori necessity characterizing mathematical and logical truths (including geometry) is that all their propositions are analytic. So, concludes Ayer, "To say that a proposition is true a priori is to say that it is a tautology" (1952: 87).

Pap develops his conception of *formal* a priori within such a logical semantic framework shaped by the contraposition between Kant and the logical empiricists. Roughly put, he builds upon Ayer's arguments while maintaining Kant's conclusion that necessary sentences don't need to be exclusively analytic.

21.3. Necessity and Analyticity

In a recent article, Limbeck-Lilienau clearly outlines Pap's perspective on necessity with the following words.

> [Pap develops] arguments against the linguistic theory of necessity, i.e., the position that all necessity is logical and that all a priori truths are analytic truths. Against this position, Pap defends a version of the synthetic a priori. […] The aim of his argument is to show that indeed there are a priori truths which can neither be reduced to logical necessity nor to analyticity. (Limbeck-Lilienau 2025: 328, 332)

Let's analyze this view thoroughly by considering the relationship between analyticity and necessity. First, it is noteworthy that Pap criticizes the semantic notion of necessity defended by logical empiricists, especially the idea that necessity can be reduced to analyticity. For Pap, the two concepts differ. He relies on Carnap's definition (1954) of an analytic sentence as "a sentence that is the *consequence* of any *sentence*" (Pap 1949/2006: 92), which makes the defined concept relative to a *given language*.

> This definition makes the defined concept […] relative to a given language (i.e., "*p* is analytic" must be regarded as elliptical[3] for "*p* is analytic in *L*"), since the syntactic concept "consequence" is defined in terms of the transformation rules for a given object-language. (Pap 1949/2006: 92)

Carnap's definition is primarily conceived for "syntactical investigations into the formal structure of artificial languages such as logical calculi and formalized arithmetic" (Pap 1949/2006: 92). However, "analytic" cannot mean "necessary" in the sense of a *syntactic* concept. A syntactic concept is defined for sentences of an uninterpreted language; these sentences cannot be said to be true or false (uninterpreted as they are), but at best (if the system is complete) it can instead be said that they are derivable from primitive sentences or refutable based on primitive sentences. What exactly does Pap reject? He rejects the reduction of necessity to analytic sentences intended as semantic sentences.

> For necessity of propositions has always been meant as a *semantic* concept: a necessary condition which any adequate analysis of "necessary" must satisfy is that the *truth-value* of a necessary proposition does not depend on any empirical facts. (Pap 1949/2006: 92)

The reasons for this rejection are implicit in Carnap's definition. First, the truth-value of any analytic sentence is ultimately factual—a claim consistent with the logical empiricist *principle of verification*. This factuality conflicts with any requirement for necessity. Second, this truth-value depends on the syntactic rules of formation and transformation that precede every semantic concept. Indeed, Carnap's definition of "analytic" in semantic terms yields a concept *corresponding* to an earlier-defined syntactic concept in the sense that:

> [...] any sentence which is analytic in the syntactic sense (e.g., "*p* or not-*p*,"[4] where the logical constants [i.e., connectives] "or" and "not" are not defined by truth-tables but occur as undefined logical symbols in the primitive sentences) becomes analytic in the semantic sense once the language to which it belongs is semantically interpreted. (Pap 1949/2006: 92)

Let's clarify these two reasons. For Carnap, analyticity requires transformation rules for a specific language. A sentence is analytic if it's "a *consequence* of another sentence or sentences" (1954: §10) under certain linguistic conditions. For instance, "Let S_1, be '(x)5 (Red(x))', and S_2 'Red(3)'; given that all positions up to 5 are red, then it's also given (implicitly) that the position 3 is red" (Ibid.). Carnap's definition makes the defined concept "relative to a given language, i.e., "*p* is analytic" implies "*p* is analytic in *L*" (Pap 1949/2006: 92), namely in a given object-language. This syntactical view of analyticity applies only to artificial languages, as their formal structure is lacking in imprecise natural languages. However, as stated in the quote above, "any sentence, which is analytic in the syntactic sense (e.g., '*p* or not-*p*')" can become "analytic in the semantic sense once the language to which it belongs is semantically interpreted" (Ibid.). Pap argues that this semantic interpretation requires a factual reference for assigning truth values. Therefore, he rejects the Carnapian definition of semantic analyticity for ignoring such a factual dependency. Pap's criticism also extends to Carnap's concept of a semantically analytic or *L*-true sentence, which is true in every state-description, like the Leibnizian concept of "truths of reason" that hold in any possible world. This form of semantic analyticity resembles syntactic analyticity because it overlooks the factual justification for the initial assignment of truth values.

Similar to Carnap, Quine defines an analytic statement as an abbreviated substitution instance of a logical principle. Let's consider a familiar formulation of a necessary proposition (hereafter P), as suggested by Pap and revised by Shieh.

P1 All sisters are female

Is "necessarily true" because it's analytic, i.e., "true by definition." Specifically, since "sister" is defined as, i.e., it's synonymous with "female sibling," and the mutual substitution of synonymous expressions doesn't change the meaning of a sentence, P1 is synonymous with.

P2 All female siblings are female

The sentence P2 represents a substitution instance of a valid schema of first-order logic (e.g., in Quine's words, a logical principle). Therefore, it's logically true.

P3 $(\forall x)(Fx.Sx \supset Sx)$

For Quine, a logical principle is defined as a true statement that contains only *logical* constants (i.e., logical connectives or operators and variables)—e.g., P3, which can be read as: for every x, F, S; "Fx and Sx" implies Sx. However, the implicit existential determination of this latter is that the sentence "something exists," formalizable as "There is an x and a P such that Px," expresses a problematic logical truth. Pap's argument can be broken down to the following claims (i.e., C1-2).

C1 The universal quantifier "$(\forall x)Px$" is equivalent to "$\sim(\exists x) \sim Px$"

C2 From the C1 equivalence, it follows that the two statements of the form "$(\forall x)Px$" and "$(\forall x) \sim Px$" are incompatible (not simultaneously true) *only if something exists*.

The statement "(∀x)Px" means "for all x, Px is true." This is a universally quantified statement asserting that every element in the domain of discourse has property P. The statement "(∀x) ~Px" means "for all x, not Px is true," or "for all x, Px is false." This is also a universally quantified statement, but it asserts that every element in the domain of discourse *does not* have property P. For these two statements to be incompatible (simultaneously true in a logical system), something must exist in the domain of discourse. Here's why:

1 If the domain of discourse were empty (i.e., nothing exists), then both "(∀x)Px" and "(∀x) ~Px" would be vacuously true. In an empty domain, there are no objects to satisfy (or fail to satisfy) the predicate P. So, the statement "for all x, Px" is true because there are no counterexamples, and similarly, "for all x, not Px" is also true. In this scenario, the two statements would not be incompatible.

2 However, if the domain of discourse is non-empty (i.e., something exists), then the two statements become incompatible. If something exists, let's call it 'a.' If "(∀x)Px" is true, then 'a' must have property P. If "(∀x) ~Px" is true, then 'a' must not have property P. An object cannot simultaneously have a property and not have that property. Therefore, in a non-empty domain, these two statements cannot both be true simultaneously.

In simpler terms, the statements are incompatible only if they can be applied to at least one thing. If there's nothing to talk about, then both statements are true. If there's something to talk about, then they can't both be true.

"And would it not be paradoxical if it depended on extralogical facts whether two given propositions are incompatible by their form?" (Pap 1949/2006: 93). Also, how can a "logical constant" be defined? A circular definition would be that logical constants are "those symbols from definitions of which the truth of logical principles follows" (Ibid.). However, this definition is unacceptable, and the problem remains since extra logical definitions unavoidably underpin logical constructions.

Nevertheless, formalization is required for evaluating the analyticity of a sentence. Pap correctly argues that a sentence with descriptive terms can be considered analytic only through its translation to a formal sentence equivalent to a logical principle.

> A statement is analytic if it is a substitution instance of a logical principle or, in case defined terms occur in it, a definitionally abbreviated substitution instance of a logical principle. This definition commits us to acceptance of an interesting consequence, whether we like it or not: if a statement, like "No part of any surface is both blue and red at the same time," contains undefined predicates ("blue," "red"), we cannot know it to be analytic unless replacement of all descriptive terms by appropriate variables leaves us with a principle of logic. (Pap 1949/2006: 94)

Moreover, even logical principles, such as "if *p*, then *p* or *q*," can be said to be their own substitution instances. When *definitional abbreviations* are spoken of, not only are definitions of descriptive constants, like "sister," referred to, but also definitions of logical constants, like "if, then." This convention enables us to say that "not (*p* and not *p*)" is a definitional expansion of "if *p*, then *p*," for example—see bivalence in note 3 above.

That said, Pap wants to prove that substitution instances, although required, don't disprove two things. *First*, an analytic principle of logic might also be necessary in the same way that synthetic propositions may likewise be necessary. Second, "*p* is necessary" doesn't entail "*p* is analytic," although the converse entailment undeniably holds.

Notwithstanding, some sentences cannot be determined from their formula, even when the descriptive constants are replaced by variables, leading to a logical truth. Pap's Fermat example isn't a fortunate one since his theorem—stating that there are no positive integers *a*, *b*, and *c* that can satisfy the equation $a^n+b^n=c^n$ for any integer value of *n* greater than 2—was proved in 1994 by Andrew Wiles. Still, his point remains valid: undecidable formulas are complicated to a degree that the formulas resulting from the formalization of controversial "necessary" statements never are.

Pap's second example is luckier. The formula corresponding to "No space–time region is both wholly red and wholly blue" would be "~(∃*x*)(∃*t*)[*Pxt*.*Qxt*]" (where surfaces constitute the range of "*x*"). As Pap argues, this formula is certainly not logically true, since we can easily find predicates which, when substituted for "*P*" and "*Q*," would yield a false statement. Let's carefully analyze this argument, as it represents a specific instance of non-analytic necessity (more below).

21.4. Temporal Statements

Pap develops two main arguments to deny the reduction of all necessary or a priori statements to analytic statements, pertaining to *temporal statements* and *the color-exclusion problem*—i.e., instances of non-analytic necessity. Let's remind that the statement's property of being *necessarily true*, which is a variation of being *more than simply true*, can be clarified by analyzing it into two distinct properties, one pertaining to the *meaning of the sentence* and another referring to *its logical truth*. Let's start by examining temporal statements.

Pap, along with C.I. Lewis, argues that analytic truth is based on certain immutable relationships between "objective meanings" that remain unaffected by accidental changes in linguistic rules. This perspective shapes his primary objection to the linguistic theory of logical necessity: he believes that the necessity of a proposition, whether it's analytic or synthetic, is a fact altogether that exists independently of linguistic conventions. However, Pap also finds views like C.I. Lewis's regarding the nature of analytic or a priori truth—where "analytic" and "a priori" are treated as synonyms—to be ultimately unsatisfactory.

For instance, about $(\forall x)(Fx.Sx \supset Sx)$, Pap argues that such an account of necessity fails to cover all truths which "we intuitively recognize as necessary" (Shieh 2006: 10), since it only satisfies the condition that necessity must retain independence from empirical facts (see Pap 1949: 92). Consider the temporal statement:

S1 If *A* precedes *B*, then *B* doesn't precede *A*

S2 $(\forall x)(\forall y)[xRy \supset \sim yRx]$

[formalization of (S1)]

As Pap comments, "I assume that few would regard this statement as factual, i.e., such that it might be conceivably disconfirmed by observations" (Pap 1949: 96). To see that it isn't analytic either, we only need to formalize the statement, obtaining "$(\forall x)(\forall y)[xRy \supset \sim yRx]$." S2 is certainly no principle of logic, nor deducible from logic. Thus, if we, nonetheless, want to call it necessary (i.e., nonfactual), we must admit that "there are necessary propositions which are not analytic" (Pap 1949/2006: 97).

As Shieh comments, Pap's problem of finding definitions to demonstrate analyticity is this: "if a definition is selected only because it allows for the derivation, from logic alone, of 'the necessary' to be established as *analytic*, then the analysis of necessity in terms of analyticity is circular" (2006: 11).

When faced with a reductive explanation of *X* in terms of *Y*, Pap examines the basis on which we identify *Y*s and finds that we do so by recognizing *X*s. Pap's conclusion is then that we have a primitive and irreducible capacity for intuitive knowledge of *X*s. (Shieh 2006: 11).

Pap strengthens his claims by using counterarguments that address potential objections. One of these counterarguments relies on the meaning of the concept "*x* precedes *y*" in logic. It generally indicates a relationship of order, where *x* precedes *y* in a specified sequence or process, whether it's a general ordering, preference, temporal sequence, or causal dependency. Indeed, "*x* precedes *y*" (i.e., $x < y$) can represent various relationships depending on the specific context. Although this ordering might not be strictly numeric and can be applied to different mathematical entities depending on the context, Pap's view of "*x* precedes *y*" can be reduced to an *ordered set* defined by the following characteristics.

Suppose *R* is a relation on set *S* satisfying three properties related to order (see Lipschutz 1998: 166–203):

O_1 *Reflexive*: For $a \in S$, we have aRa.

O_2 *Antisymmetric*: If aRb and bRa, then $a=b$.

O_3 *Transitive*: If aRb and bRc, then aRc.

If $O_{1\text{-}3}$ are satisfied, then *R* is a *partial order*, or simply, an *order relation*, where *R* defines a *partial ordering* of *S*. Consequently, the set *S* characterized by the partial ordering *R* is called a *partially ordered set*, or simply an *ordered set* (also known as *poset*). From $O_{1\text{-}3}$ properties, we can define the standard order relation, i.e., the *usual order*. The usual relation $\leq$ (read "less than or equal to") is generally defined on the positive integers P or on any subset of the real numbers R. Thus, a partial ordering relation—denoted by $\preccurlyeq$—redefine the above $O_{1\text{-}3}$ properties of a partial order in this form:

O_1 *Reflexive*: For $a \in S$, we have $a \preccurlyeq a$.

O_2 *Antisymmetric*: If $a \leqslant b$ and $b \leqslant a$, then $a=b$.

O_3 *Transitive*: If $a \leqslant b$ and $b \leqslant c$, then $a \leqslant c$.[5]

Hence, a relation *R* on a set *S* is a *partial ordering* or a *partial order* of *S* if and only if *R* is reflexive, antisymmetric, and transitive. Pap builds on the concept of partial ordering to illustrate examples of non-analytic necessity.

Consequently, Pap tries to counterargue objections against those instances by arguing that even though a statement like "$(\forall x)(\forall y)[xRy \supset {\sim}yRx]$" isn't factual, it doesn't imply that it isn't synthetic either. As a general criterion for evaluating a sentence, we must reach its *ultimately simple* concepts, as Carnap suggests. If we apply this criterion to the relationship of temporal succession, we obtain simple concepts, whose combination leads to a proposition that remains true regardless of empirical facts—it cannot be disproven by observations. Nonetheless, this necessary proposition, despite being nonfactual, *may still be* synthetic. Therefore, the term "analytic" will not be used as an *analysans*[6] for "necessary." Roughly put, analyticity and necessity don't need to overlap.

As Pap argues, a non-solution would be trying to prove the necessary proposition "$(\forall x)(\forall y)[xRy \supset {\sim}yRx]$" using logic alone. This attempt would beg the question by assuming what it's aiming to prove; any attempted proof would need a definition of "*x* precedes *y*" from which the asymmetry of this temporal relation could be formally deduced. However, if this definition is deemed sufficient or adequate, what would be the standard for its adequacy? A definition of "*xPy*" (symbolizing "*x* precedes *y*") could only be adequate if it leads to the conclusion that *P* is asymmetric, namely solely by entailing the asymmetry of *P*. Let's briefly recall what "symmetric" means.

1 In logic, a relation is *symmetric* if it's identical with its converse (e.g., "is a sibling of"); otherwise, it's *non-symmetric*. A *non-symmetric* relation includes relations that are "partially symmetric" or "not symmetric at all," i.e., *asymmetric*. An *asymmetric* relation excludes its converse, representing a more restrictive type of non-symmetry (e.g., "is the father of").

In discrete mathematics, a relation *R* on a set *A* is symmetric if whenever *aRb* then *bRa*, that is, if whenever $(a, b) \in R$, then also $(b, a) \in R$—(b, a) being the converse of (a, b). Using logical notation, we can write it as: $\forall a, b \in A \ ((a, b) \in R \rightarrow (b, a) \in R)$. In contrast, *R* is not symmetric if there exists $(a, b) \in A$ such that $(a, b) \in R$ but $(b, a) \notin R$.

Let's recall its difference with the antisymmetric property: a relation *R* on a set *A* is antisymmetric if whenever *aRb* and *bRa* then $a=b$, that is, if whenever (a, b) and (b, a) belong to *R* then $a=b$; *R* is not antisymmetric if there exists $(a, b) \in A$ such that (a, b) and (b, a) belong to *R*, but $a \neq b$.

Therefore, the criterion of adequacy for temporal relations is the asymmetry through which we can formally prove the relation "*xPy*." However, Pap argues that simply listing all the formal properties that P must possess does not lead to a sufficiently specific definition that can distinguish P from other formally similar relations. For instance, "If I define *P* as *asymmetrical*, *irreflexive*, and *transitive*, the relation expressed by '*x* is greater than *y*,' as holding between real numbers, would also satisfy the definition [of '*xPy*']" (Pap 1949/2006: 97). As before, let's clarify the meaning of the irreflexive and transitive relations discussed in Pap's analysis.

2 A logical relation is *reflexive* if every element within its domain is related to itself (e.g., "is contemporaneous with"); otherwise, it's *non-reflexive* (e.g., "is the teacher of"). A non-reflexive relation is *irreflexive* if it excludes identity (e.g., "is the father of").

In discrete mathematics, a relation *R* on a set *A* is reflexive if *aRa* for every $a \in A$, that is, if $(a, a) \in R$ for every $a \in A$ (for each $a \in A$ there is $(a, a) \in R$). *R* is not reflexive on a set *A* if there exists any (i.e., at least one) $a \in A$ such that $(a, a) \notin R$. *R* is irreflexive on a set *A* if for all $a \in A$, $(a, a) \notin R$.

3 A logical relation is *transitive* if whenever element *a* is related to *b* and *b* is related to *c*, then *a* is also related to *c* (e.g., "is an ancestor of"); otherwise, it's *non-transitive* (e.g., "is a friend of"). Broadly conceived, a non-transitive relation is *intransitive* if it never holds for the next member and the one after it (e.g., "is the father of").

In discrete mathematics, a relation R on a set A is transitive if whenever aRb and bRc then aRc, that is, if whenever $(a, b)\in R$ and $(b, c)\in R$ then $(a, c)\in R$. In contrast, R is not transitive if there exists $(a, b, c)\in A$ such that (a, b), $(b, c)\in R$ but $(a, c)\notin R$.

Pap offers a solution to the previous impasse. A simple way to achieve uniqueness and satisfactorily specify "xPy" is to add the condition, "The field of P consists of events." With this definition, the necessary proposition S1 is reduced to a substitution instance of the logical truth, "$xPy.[(xPy \supset {\sim}yPx).Q] \supset {\sim}yPx$" where "$Q$" represents the remaining defining conditions for the use of "P". However, Pap argues, it's obvious that "acceptance of the definition—from which the asymmetry of temporal succession has thus been deduced—presupposes acceptance of the very proposition 'Temporal succession is asymmetrical' as self-evident" (1949/2006: 98). This leads to a circular definition that reveals the fallacy of an argument that assumes the very thing it's trying to prove is already true. As Pap concludes, "This way of proving that the debated proposition is, in spite of superficial appearances, analytic, is therefore grossly circular" (Ibid.).

21.5. The Color-Exclusion Problem

A second argument demonstrating instances of non-analytic necessity relates to *the color-exclusion problem*. Pap presents this idea clearly. The statement "No space–time region is both wholly blue and wholly red" (1949/2006: 98) is a necessary truth, but that doesn't imply it's also analytic. Pap also explores related statements such as, "Nothing can be simultaneously blue and red all over," and "Whatever is colored is extended" (Ibid.). He aims to prove that the "${\sim}(\exists x)(\exists t)(\text{blue}_{xt}.\ \text{red}_{xt})$" statement isn't a logical deduction. His central tenet is as follows.

> Indeed, if "blue" and "red" designate unanalyzable qualities, it is difficult to see how analysis could ever reveal that this statement is a substitution instance of a logical truth. (Pap 1949/2006: 98)

Pap reads the statement "${\sim}(\exists x)(\exists t)(\text{blue}_{xt}.\ \text{red}_{xt})$" by identifying the type of entities to which colors can apply. These entities are surfaces, regardless of whether they may be located in physical or perceptual space. He then develops a related argument as follows:

> [...] if I say "x is blue at t" and also "y is red at t" (where the values of the variables "x" and "y" are names of surfaces), I have already implicitly asserted "$x \neq y$"; in the same way as simultaneous occupancy of different places is tacitly regarded as a criterion of the presence of *different* things at those places. (Pap 1949/2006: 98)

Therefore, arguments of this nature are circular. If we formalize the proposition above, we arrive at the claims:

1	$(\forall x)(\forall y)(\forall t)(\text{if blue}_{xt}.\ \text{red}_{xt}, \text{then } x \neq y)$	let's call it T
2	${\sim}(\exists x)(\exists t)(\text{blue}_{xt}.\ \text{red}_{xt})$	let's call it S

It follows that S derives from T. Hence, we might (mistakenly) assert that "if T, then S" is analytic. However, Pap counters this argument by stating that:

> [...] we would first have to prove that T is analytic before we could assert that S is. As I follow the rule, "Any proposition is to be held synthetic unless it is derivable from logic alone," I hold T to be synthetic until such time as conclusive proof of the contrary is produced. (Pap 1949/2006: 98)

For Pap, the only meaning we can attach to the statement "'non-blue' is part of the meaning of 'red'" is just "'x is red at t' *entails* 'x is not blue at t'." The question at issue is whether this entailment can be regarded as analytic (or formal). It's clear that "non-blue" cannot be seen as an element of the concept of "red" in the same way that "male" is an intrinsic part of the concept of "father."

Similarly, "If x is colored, then x is extended" presents the same issue: "the involved predicates cannot be analyzed in such a way as to transform the propositions into tautologies; they can only be ostensively defined" (Pap 1949/2006: 98). This means defining by direct demonstration, such as pointing to an example. However, the question remains: why is this particular argument not considered analytic? Below is an explanation of the corresponding argument for analyticity.

(a) If we stipulate that only names or descriptions of surfaces are admissible values of "x" in the universal statement "if *x* is colored, then *x* is extended."

(b) Also, each substitution instance of "*x*" is analytic, since it has the form, "if this surface is colored, then it is a surface," which is reducible to, "if this is a surface and colored, then it is a surface" (by logical simplification).

(c) Therefore, the universal statement itself, which may be interpreted as the logical product of all its substitution instances, must be analytic by default.

Pap objects that such an argument begs the question. The statement, "*only surfaces can significantly be said to have a color*," differs in an important respect from statements like "*only animals (including human beings) can significantly be called fathers*." The latter involves analyzable predicates and could be replaced by the statement, "fathers are defined as a subclass of human beings." At the same time, we cannot assume, without begging the very question at issue, that "*x* is colored" *entails by definition* "*x* is a surface" (see Pap 1949/2006: 100).

However, what does Pap precisely object to? The argument for the necessary proposition "if *x* is colored, then *x* is extended" is analytic. Pap doesn't deny that the analysis of "*x* is colored," deducing the consequent from the antecedent, relies on an analytic connection. He contends "only that there are necessary propositions, i.e., propositions which are known to be true independently of empirical observations, which are not *known* to be analytic" (1949/2006: 100). Therefore, Pap rejects the tenet of logical empiricism that all necessary propositions are known to be analytic (Ibid.). Indeed, according to Pap, the standard argument for analyticity implicitly assumes an initial condition, a stipulation that compromises its validity. Indeed, suppose we stipulate in advance that an analysis of the antecedent will be correct only if it enables deduction of the consequent. In that case, it's not surprising that any correct analysis of the concept in question will reveal the analytic nature of the statement.

> [...] "if $x = y$, then $y = x$," is necessary; quite independently of our knowledge of logic, one feels that it would be self-contradictory to deny any substitution instance thereof. But as long as the relation of identity of individuals remains unanalyzed, there is no way of deducing it from logic. "If *xRy*, then *yRx*" is not true by its form. (Pap 1949/2006: 100)

Thus, Pap concludes, nobody "will be perfectly safe in claiming that this proposition [i.e., "if *xRy*, then *yRx*"] will turn out to be analytic once the involved relation is correctly analyzed" (1949/2006: 100). Do you remember the symmetric property of relations? *R* is symmetric if and only if for every *x* and *y*, if *xRy*, then *yRx*—i.e., $(\forall x)(\forall y)[xRy \supset yRx]$, and *R* is asymmetric if and only if for every *x* and *y*, if *xRy* then ~*yRx*—i.e., $(\forall x)(\forall y)[xRy \supset {\sim}yRx]$. However, nothing in the logical form of *xRy* contains the implication *yRx* or ~*yRx*.

> The unanalyzed proposition $\forall x(\text{Red}(x) \rightarrow \neg\text{Blue}(x))$ is obviously not a logical truth. It can only be shown to be analytic if we can give an adequate analysis of its descriptive color terms. (Limbeck-Lilienau 2025: 333)

As Limbeck-Lilienau reminds us, in this context, Pap adopts Quine's conception of analyticity (Pap 1955: 202), which depends on two criteria.

1. A proposition is analytic if its truth is based on its logical signs alone. This is the case for logical truths such as tautologies or the laws of logic, which are true independently of the meaning of their non-logical terms.

2. A proposition is analytic if, via an analysis of its descriptive terms, the proposition can be transformed into a logical truth. For example, the proposition "All bachelors are unmarried" can be transformed into the logical truth "All unmarried men are unmarried" by analyzing "bachelor" as "unmarried man" and by substituting the former with the latter.

Pap explores another solution for color concepts, such as “Red,” which he considers *simple* and, therefore, *unanalyzable*. To support this thesis, he presents a proof by contradiction. So, let’s see what happens if we try to transform color concepts into analytic propositions by analyzing “Red” through the negation of all other colors:

> “Red” $=_{Df}$ ¬Blue ∧ ¬Green ∧ ¬Yellow ∧ ¬Black, and so on ad infinitum.”

This analysis would remain incomplete by default because it involves infinite *definientia*. Also, it would be circular since “Blue” would itself have to be analyzed into “¬Red ∧ ...”. “So, unless a more adequate analysis of color terms can be given, the proposition ∀*x*(Red(*x*) → ¬Blue(*x*)) cannot be shown to be analytic. Nevertheless, it still seems to be a necessary proposition” (Limbeck-Lilienau 2025: 333).

The same issue arises in Wittgenstein’s *Tractatus*. Although color concepts are regarded as *unanalyzable*, the statement “Red” seems to imply “non-Blue.” Color sentences are elementary propositions. Being elementary means that “there can be no [other] elementary proposition contradicting it” (*Tractatus* 4.211). Elementary propositions are mutually independent and cannot determine the truth or falsity of another nor can “be deduced from another” (*Tractatus* 5.134). This *logical independence* mirrors Wittgenstein’s *logical atomism*, which asserts that atomic states of affairs are reciprocally independent. Consistently, the truth or falsity of one elementary proposition provides no information about the truth or falsity of any other. However, Wittgenstein also claims that “the simultaneous presence of two colours at the same place in the visual field is impossible, in fact logically impossible, since it is ruled out by the logical structure of colour” (*Tractatus* 6.3751). Here is Pap’s issue anew. If the term “Red” cannot be analyzed further, then the statement “*x* is red” qualifies as an elementary proposition. However, due to its logical nature, this color statement seems to exclude other elementary propositions, such as “*x* is blue” or “*x* is non-blue.” In fact, “the statement that a point in the visual field has two different colours at the same time is a contradiction” (*Tractatus* 6.3751). As Limbeck-Lilienau concludes, “Wittgenstein had to concede that though “Red” is not analyzable, it still excludes “Blue” (and other colors)” (2025: 333).

We also encounter impossible combinations when using standard truth tables, where each line represents a possible state of the world, to evaluate color propositions. For example, let’s *p* be “*a* is red” and *q* be “*a* is blue,” where *a* designates the same spatiotemporal area. The distribution of truth-values for “*p* ∧ *q*” shows that their coexistence is possible, namely true (see the truth tables below).

		Conjunction			Conditional	
p T T F F	q T F T F	p∧q T F F F	p T T F F	q T F T F	p→q T F T T	p→¬q F T T T

However, it is impossible for “*p* and *q*” to both be true, for “*p*: T and *q*: F” or “*p*: F and *q*: T” to be false, and for “*p*: T truly implies non-*q*: F” to be false either. This impossibility could be solved by a complete “logical syntax,” which could exclude similar cases by some rule (Waismann 1940).

> Our logic allows us to say that “a is red” and “a is blue” at the same time. But a more complete logic should exclude this. (Limbeck-Lilienau 2025: 334)

Hence, Pap concludes that the statement ∀*x*(Red(*x*) → ¬Blue(*x*)) is necessary without being a tautology. Therefore, its necessity is a priori but isn’t analytic.

21.6. The Material a Priori

A reasoning similar to Pap’s can also be found in Husserl’s defense of the *material* a priori. Husserl and other phenomenologists partially disagree with Kantian formalism, according to which the a priori is a strictly formal notion. Kant denies any room for materiality in the a priori and reduces all aspects of apriority to the forms of our knowledge alone (see Parrini 2002: 136). Logical empiricists align with Kant, particularly Schlick (1930), who openly critiques Husserl’s idea (see

Lanfredini 2006). Notwithstanding this rejection by logical empiricism, Pap retains some elements of the material a priori.

Let's clarify that both Husserl and Pap share the belief with Kant and Schlick that a priori propositions don't require validation from experience, while a posteriori propositions do. Furthermore, they argue that *necessity* and *universality* distinctly characterize a priori propositions. However, there is disagreement among them regarding the boundaries of the a priori. Kant's concept of *synthesis* a priori is largely rejected, and Husserl's idea of *material* a priori is particularly criticized by those who equate *apriority* with *analyticity*, such as the logical empiricists, including Schlick, Carnap, among others.

Husserl argues that some *synthetic* a priori propositions are empirical or factual because their justification relies on factors that are not entirely pure or a priori—see Simons's example below. This view positions him apart from Kant. A notable defense of Husserl's position is presented by Da Silva (2017) and Simons (1992), whose main arguments can be presented as follows (see Oliva 2024).

First, analytic propositions are true a priori by default. However, analyticity is not defined solely by the containment of the predicate class within the subject class; analytic truths are not considered true only because of the meanings of their constitutive terms. According to Husserl, "analytic propositions are those whose logical value is preserved under formalization," where formalization means substituting variables for names (Da Silva 2017: 98).

Kant and Husserl see apriority differently. For Kant, synthetic a priori truths reveal the structural or necessary properties of experience and the objects of experience, defining the *possibility* of both. Dissimilarly, Husserl holds that "propositions whose *complete* formalization doesn't preserve logical value are synthetic" (Da Silva 2017: 98). Da Silva further explains that "conceptual truths are *analytic*" when "the scopes of the concepts involved extend to the domain of *all* object[s]" (Ibid.). Nevertheless, there may be restrictions that apply, which lead to Husserl's conception of material a priori.

> Some propositions, however, preserve truth-value under restrict[ed] formalization, that is, their logical value is preserved *provided* the variables are confined to more restrict[ed] domains of variability. If we think of these domains as extensions of concepts, synthetic truths are conceptual truths involving at least one concept whose scope is restricted to a proper subdomain of the domain of all objects [...]. (Da Silva 2017: 98)

The key feature of Husserl's restriction pertains to the domain of justification, namely an a priori specification of the logical universe. Accordingly, formal concepts, such as "number" and "whole," are broad and lack specific (i.e., material) content because they encompass all objects without distinction. In contrast, material (i.e., restricted) concepts gain meaning by specifying the domain of their extension. Therefore, particular domains are inherently material, and the laws related to them are synthetic. Consequently, material concepts are considered a posteriori when the truths they express are *contingent*, but they are a priori when these truths are *necessary*.

In this regard, Simons points out that a statement like "Nothing can be simultaneously red and green" isn't explicitly analytic according to Husserl (1992: 374). In fact, if we were to replace "green" with another quality, such as "round" or "soft," the statement would become false. Moreover, it would also be implausible to argue that the concept of "red" inherently excludes "green;" therefore, these propositions cannot be considered implicitly analytic either (see Simons 1992: 375).

In conclusion, Husserl argues that when we consider a part-whole relationship (which is entirely formal), such as $P = (x, y)$, specifying the parts introduces certain restrictions on it. For example, if we define these parts (i.e., variables) as *red* and *green*, their association under P (that stands for any class whatsoever) would only be valid if other conditions are met. Specifically, if x represents *all red* objects and y represents *all green* objects, then $P = (x, y)$ at time t^n holds true only if n refers to any time point where these colors are not simultaneously present. If this is correct, Husserl's concept of the *material* a priori could represent a particular formalization constrained by factual implications, designating a specific area, such as a regional subfield, of special apriority. Accordingly, Husserl envisions various distinct material a priori ontologies.

21.7. Final Remarks

Let's briefly summarize what has been accomplished. The framework of Pap's semantic analysis is shaped by *logical* *empiricism* and language philosophy, primarily relying on the verifiability principle. In this context, Pap aims to challenge the idea that necessary statements can be reduced to analytic statements by default.

Analyticity and *apriority* have been central topics in the works of Leibniz, Kant, the logical empiricists, Wittgenstein, and philosophers of language (particularly Quine, who critiques analyticity). For Leibniz, necessary truths are examples of identities where the conceptual classes (the S-term and the P-term) are related by inclusion and can be analyzed accordingly. Kant expands the concept of necessity, which Leibniz restricted to analytic sentences, to include non-analytic instances, such as the well-known cases of synthesis a priori. In contrast, Ayer defines analytic sentences as those whose validity relies solely on the definitions of the symbols they contain. For instance, logical and mathematical truths are considered necessary because they lack factual content. These truths are analytic, as their validity isn't determined by empirical facts, unlike synthetic sentences. According to Ayer, if the validity of proposition *p* depends on *x*, then it's necessary to refer to *x* to verify *p*. In the case of analytic sentences, demonstrating their truth value requires no appeal to anything other than the meanings of their terms.

Pap's primary objective is to defend a version of the *synthetic* a priori, arguing that there are a priori *truths* that cannot be reduced to either logical necessity or analyticity. His strategy involves critiquing Carnap's concept of analyticity, which is based on *syntactic* concepts. According to Pap, "analytic" cannot simply mean "necessary" in the context of a *syntactic* definition. Syntactic concepts apply to sentences in an uninterpreted language, meaning these sentences cannot be classified as true or false because they lack interpretation. In contrast, the condition for the necessity of propositions should rely on a *semantic* concept. However, this necessary condition doesn't depend on empirical facts. To support his reasoning, Pap discusses two examples of non-analytic necessity, providing their context first.

Consider the statement "(∀x)Px." This universal statement asserts that "for all x, Px is true," meaning that every element in the domain of discourse has a certain property P. In contrast, the statement "(∀x) ~Px" means "for all x, Px is non-true, i.e., false." This is also a universally quantified statement, but it contradicts the previous one, making the two statements incompatible. To establish their truth value, there must be something in the domain of discourse that allows us to construct an adequate truth-table. Pap makes this point very clearly.

> Is it my contention, then, that even a "formal" statement, as it would commonly be called, like, "for any *x* and *y*, if *x* = *y*, then *y* = *x*," is a *synthetic a priori* truth? This would, indeed, amount to going more Kantian than Kant himself; for, on the same principle, it could be argued that all logical truths, which Kant at least conceded to be analytic, are synthetic. Take, for example, the commutative law for logical conjunction, just mentioned. Obviously, I cannot prove that "(*p* and *q*) ≡ (*q* and *p*)" is tautologous, unless I first construct an adequate truth-table defining the use of "and." But surely one of the criteria of adequacy for such a truth-table definition consists in the possibility of deriving the commutative law as a tautology. If, for example, a "T" were [conventionally] associated with " *p* and *q*" when the combination "F T" holds, and an "F" when the combination "T F" holds, the resulting definition would be rejected as inadequate just because it would entail that the commutative law is not a tautology. (Pap 1949/2006: 101)

The first case of non-analytic necessity involves *temporal statements*, such as "If *A* precedes *B*, then *B* doesn't precede *A*." In symbols, this can be expressed as "(∀*x*)(∀*y*)[*xRy* ⊃ ~*yRx*]." Pap maintains that we have a primitive and irreducible capacity for intuitive knowledge of these types of sentences. Indeed, "*x* precedes *y*" can represent various relationships depending on the specific context. Pap's central thesis is that although a statement like "(∀*x*)(∀*y*)[*xRy* ⊃ ~*yRx*]" isn't factual, this doesn't mean it isn't synthetic either. In fact, the criterion for adequacy in temporal relations is the asymmetry that allows us to formally prove the relation "*xPy*."

The second case pertains to *the color-exclusion problem*. Pap argues that the statement "No space–time region is both wholly blue and wholly red" (1949/2006: 98) is a necessary truth, but not analytic. After he formalizes the statement as "~(∃*x*)(∃*t*)(blue$_{xt}$. red$_{xt}$)," Pap notices that the truth values of it depends on identifying what type of entities are involved. The entities to which colors can apply are surfaces, whether they exist in physical or perceptual space. The meaning of these entities reveals the circularity of that analytic sentence. Indeed, "if I say '*x* is blue at *t*' and also '*y* is red at *t*' (where the values of the variables 'x' and'y' name surfaces), I have implicitly asserted that '$x \neq y$'" (Pap 1949/2006: 98). Therefore, arguments of this nature are circular.

However, they represent cases of non-analytic necessity. Is "non-blue" part of the meaning of "red"? The statement suggests that "'x is red at t' *entails* 'x isn't blue at t'," which is an entailment that Pap denies. Consequently, he concludes that the statement "$\forall x(\text{Red}(x) \rightarrow \neg\text{Blue}(x))$" is necessary without being a tautology. Thus, this type of necessity, although a priori, isn't analytic.

Support for Pap's analysis can be found in Husserl's concept of *material* a priori, which refers to a specific formalization that is justified by its factual implications. By limiting the scope of apriority, Husserl identifies a particular domain of special apriority, aligning it with Pap's instances of non-analytic necessity.

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1
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2
Pap's philosophical style of thinking is highly analytic. In *Semantics and Necessary Truth*, he states, "very few definitive conclusions are reached in this book, that perhaps more problems have been formulated than have been solved" (1958: xiv)." Nonetheless, Pap's writings are marked by two main themes: (1) a radical anti-reductionism, which critiques the efforts of logical empiricists and philosophers of language to dismiss any metaphysical categories, along with (2) a related intuitionism.

3
In a logical context, "elliptical" describes a type of reasoning or argument that is incomplete or leaves out certain steps or factors, making it difficult to fully grasp the underlying logic. It's like a statement with missing words, where the intended meaning is understood by the listener or reader based on context.

4
The *principle of bivalence* asserts that every declarative proposition must be assigned one and only one truth value, either true or false. In formal logic, one way to express this principle is by incorporating both the law of excluded middle and the law of non-contradiction. The formula of logical bivalence is: $\forall p\ [(p \vee \neg p) \wedge \neg(p \wedge \neg p)]$. This states that for every proposition p, either p is true, or its negation is true (law of excluded middle), and p cannot be both true and false (law of non-contradiction).

5
To clarify, an ordered set consists of two components, a set S and the partial ordering $\preccurlyeq$. Notwithstanding, if the partial ordering is clear from the context of our discussion, we can simply refer to the order set S as S; otherwise, we must denote the ordered set using the pair $(S, \preccurlyeq)$. Further, if S is an order set, then the statement $a \preccurlyeq b$ is read "a precedes b." In this context, we also differentiate between the following kinds of order relations: 1) $a \prec b$

means $a \preccurlyeq b$ and $a \neq b$ (i.e., "*a* strictly precedes *b*"); 2) $b \succcurlyeq a$ means $a \preccurlyeq b$ (i.e., "*b* succeeds *a*"); 3) $b \succ a$ means $a \prec b$ (i.e., "*b* strictly succeeds *a*"). Finally, the symbols $\npreceq$, $\nprec$, $\nsucceq$, and $\nsucc$ are self-explanatory.

6 "Analysans" or "definiens" refers to that which is used to explain something else—i.e., the analysandum or definiendum – through "analysis" or "definition."